CHESS DOMINATION PROBLEMS

Karel PASTOR
Palacký University Olomouc, Faculty of Education (Czech Republic)
karel.pastor@upol.cz

Abstract

It is very important to develop combinatorial skills of pupils aged 6-11. A board game can be attractive and useful tool to achieve this goal. Chess is one of the most popular games, more than 600 million people around the world know chess rules. Combinatorial skills can be developed not only by playing chess games or by solving chess diagrams, but also by solving some mathematical chess problems. We will focus on the domination problems in our paper. In contrast to the classic chessboard 8×8, we will use a smaller 4×4 and 6×6 chessboard, respectively, to make chess domination problems more accessible to pupils aged 6 to 11.

Keywords: Mathematical chess problems, domination problems, Mathematical Kangaroo.

1. Introduction

We recall ("Mathematical chess problems", Wikipedia) that a mathematical chess problem is a mathematical problem that is formulated using a chessboard and chess pieces. Chess mathematical problems have been already studied by Thabit ibn Qurra (836-901), Arabian astronomer and mathematician ("Thabit ibn Qurra", Wikipedia). We note that Thabit ibn Qurra discovered equation for determining amicable numbers. Amicable numbers are two different numbers so related the sum of the proper divisors of each is equal to the other number – for example 220 and 284 are amicable numbers ("Thabit ibn Qurra", Wikipedia). Later, also next famous mathematicians as for example Adrien-Marie Legendre (1752-1833) or Carl Friedrich Gauss (1777-1855) have studied mathematical chess problems ("Mathematical chess problems", Wikipedia).

Domination problems belong among mathematical chess problems. In these problems it is requested to find a minimum number of pieces of the given kind and place them on a chess board in such a way, that all free squares of the board are attacked by at least one piece ("Mathematical chess problems", Wikipedia).

Domination chess problems have been resolved generally for the chessboard n×n only for the following chess pieces: king, rook and bishop. When it comes to queen and knight, the situation is clear only for small n (Chybová, 2017).

In our paper, we will reduce the classic chessboard 8×8 to a smaller 4×4 and 6×6 chessboard, respectively, in order to make chess domination problems more accessible to pupils aged 6 to 11.

The movements of classical chess pieces (king, queen, rook, bishop and knight) can be found for example in ("Rules of chess", Wikipedia). We recall only the movement of a special piece named kangaroo that was introduced in Mathematical Kangaroo ("Matematický klokan, 2015").

A kangaroo moves three squares horizontally then one square vertically, or one square horizontally then three squares vertically, see Figure 1.

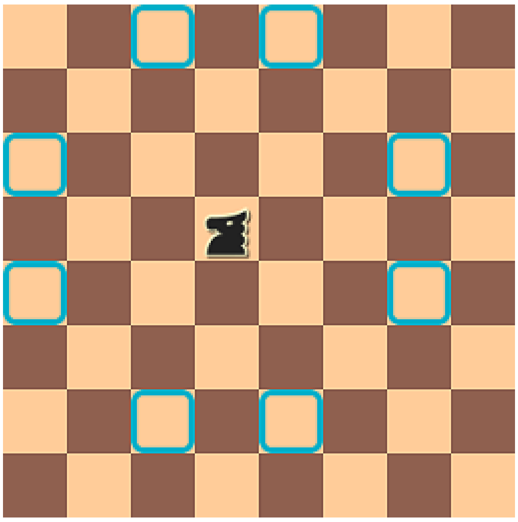

Figure 1. Moves of a kangaroo

2. 4×4 and 6×6 chessboards

In this section, we will show the possible solutions of chess domination problems for the 4×4 and 6×6 chessboards.

A. King

The minimum number of kings, which can be placed on an *n*×*n* chessboard so that all free squares of the board are attacked by at least one king, equals to

$$\left\lfloor \frac{n+2}{3} \right\rfloor^2,$$

where $\lfloor x \rfloor$ denotes the integer part of x (Chybová, str. 35, 2017). Therefore, we have that the minimum number of kings for the 4×4 chessboard equals 4, and the minimum number of kings for the 6×6 chessboard also equals 4. See Figures 2 and 3, respectively.

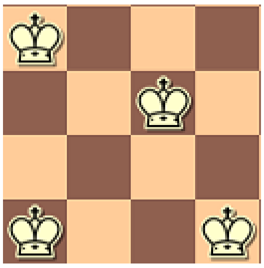

Figure 2. Kings and 4×4

B. Queen

The minimum number of queens, which can be placed on an *n*×*n* chessboard so that all free squares of the board are attacked by at least one queen, is known only for small natural numbers, see (Chybová, str. 44, 2017). The minimum number of queens for the 4×4 chessboard equals 2 and the minimum number of queens for the 6×6 chessboard equals 3. See Figures 4 and 5, respectively. The solution given in Figure 5 was obtained by the symmetry of chessboard from the example presented in (Chybová, str. 45, 2017).

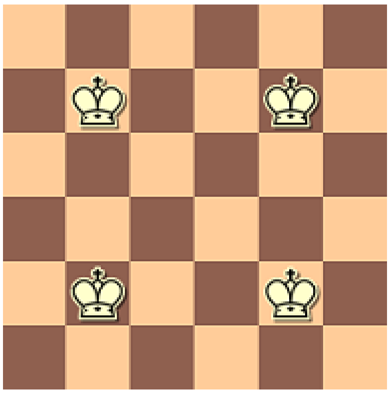

Figure 3. Kings and 6×6

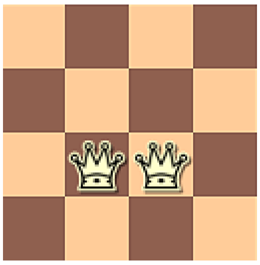

Figure 4. Queens and 4×4

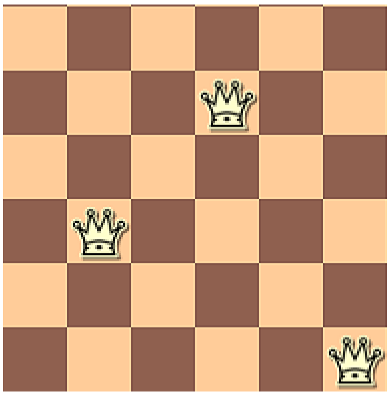

Figure 5. Queens and 6×6

C. Rook

The minimum number of rooks, which can be placed on an $n \times n$ chessboard so that all free squares of the board are attacked by at least one rook, equals to n (Chybová, str. 35, 2017). Thus, we have that the minimum number of rooks for the 4×4 chessboard equals 4, and the minimum number of rooks for the 6×6 chessboard equals 6. See Figures 6 and 7, respectively.

D. Bishop

The minimum number of bishops, which can be placed on an $n \times n$ chessboard so that all free squares of the board are attacked by at least one bishop, equals to n (Chybová, str. 36, 2017). Thus, we have that the minimum number of bishops for the 4×4 chessboard equals 4, and the minimum number of bishops for the 6×6 chessboard equals 6. See Figures 8 and 9, respectively.

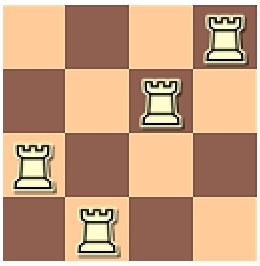

Figure 6. Rooks and 4×4

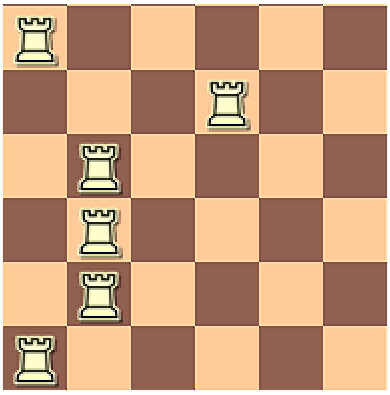

Figure 7. Rooks and 6×6

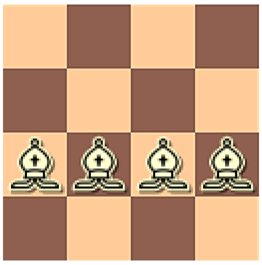

Figure 8. Bishops and 4×4

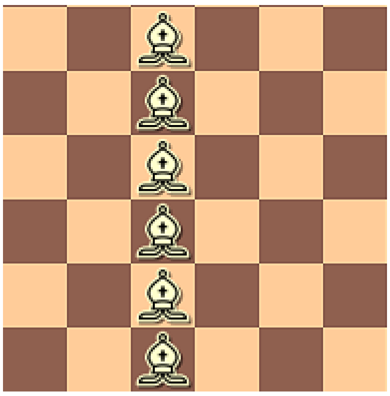

Figure 9. Bishops and 6×6

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E. Knight

The minimum number of knights, which can be placed on an *n*×*n* chessboard so that all free squares of the board are attacked by at least one knight, is known only for small natural numbers, see (Chybová, str. 46, 2017). The minimum number of knights for the 4×4 chessboard equals 4 and the minimum number of knights for the 6×6 chessboard equals 8. Examples which are given on Figure 10 and Figure 11, respectively, were presented in (Chybová, str. 46, 2017).

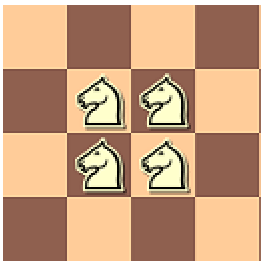

Figure 10. Knights and 4×4

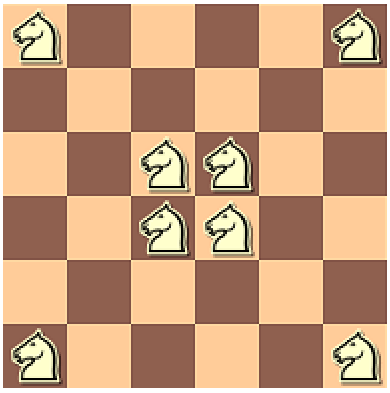

Figure 11. Knights and 6×6

Finishing this section, we will solve the domination problem for the kangaroo and the 4×4 and 6×6 chessboards.

i) 4×4 chessboards

An interior square of 4×4 chessboard cannot be attacked by any kangaroo, therefore we must place kangaroos on the all four interior squares of 4×4 chessboard. Since a kangaroo on an outer square of 4×4 chessboard attacks just two outer squares, it is necessary to use minimally 4 (=12:3) kangaroos on outer squares. Figure 12 illustrates that the number 4 of kangaroos on outer squares is enough.

Therefore, the minimum number of kangaroos, which can be placed on the 4×4 chessboard so that all free squares of the board are attacked by at least one kangaroo, equals to 8.

Figure 12. Kangaroos and 4x4

ii) 6×6 chessboards

A kangaroo can attack maximally 4 squares of 6×6 chessboard. Because 36:5=7.2, at least 8 kangaroos must be placed on the 6x6 chessboard minimally. Figure 13 shows that the number of 8 kangaroos is enough.

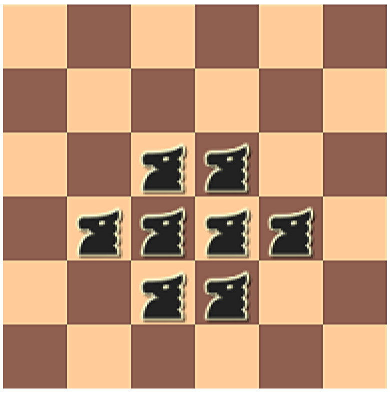

Figure 13. Kangaroos and 6x6

3. Conclusion

We were interested in some chess dominance problems that can be solved by pupils aged 6 to 11. A special attention was focused on a special piece named kangaroo.

To solve the previous problems pupils can use, for example, a printed chessboard and beans instead of pieces, or they can use some computer application, see e.g. ("Chess Diagram Setup").

Some of the previous problems are very simple (such as the problem of rooks associated with Figure 6) and pupils can solve them very quickly.

Some problems may be more difficult to some pupils aged 6-11 (such as the problem of kangaroos associated with Figure 13). In this case, the teacher (coach, parent) has the possibility to give some help - for example by placing one or more pieces on the chessboard.

Another kind of mathematical chess problems is an independence problem ("Mathematical chess problems", Wikipedia). In these problems it is requested to find a maximum number of pieces of the given kind and place them on a chess board so that none of the pieces attacks each other. In (Pastor, 2019), we have tried to make chess independence problems more accessible to pupils aged 6 to 11.

References

Chybová, L. (2017). *Šachové úlohy v kombinatorice.* Diplomová práce (in Czech). Praha: Univerzita Karlova. Retrieved 10-03-2020, from http://kdm.karlin.mff.cuni.cz/diplomky/lucie_chybova_dp/sachove-ulohy.pdf

Pastor, K. (2019). Chess independence problem. *Elementary Mathematics Education Journal 1 (2).* Retrieved 10-03-2020, from http://emejournal.upol.cz/Issues/Vol1No2/Pastor_2019_Vol1No2.pdf

(n.d.). Amicable numbers. (Wikipedia). Retrieved 10-03-2020, from https://en.wikipedia.org/wiki/Amicable_numbers

(n.d.). Mathematical chess problems. (Wikipedia). Retrieved 10-03-2020, from https://en.wikipedia.org/wiki/Mathematical_chess_problem

(n.d.). Thabit ibn Qurra. (Wikipedia, in Czech). Retrieved 10-03-2020, from https://cs.wikipedia.org/wiki/Thabit_ibn_Qurra

(n.d). Matematický klokan 2015 (in Czech). Retrieved 10-03-2020, from http://www.matematickyklokan.net/phocadownload/sborniky/sbornik_klokan_2015.pdf

(n.d.). Chess Diagram Setup. Retrieved 10-03-2020, from https://www.jinchess.com/chessboard/composer/